On Non-Linear operators for Geometric Deep Learning

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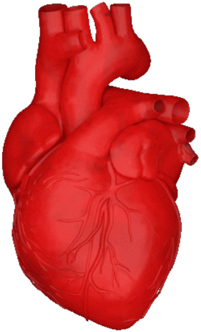
Overview of the result

First characterizations of:

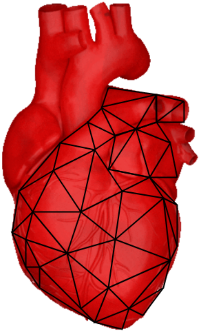
* **equivariant networks** to space of transformations with **infinite** number of dimensions.

* **the biggest possible set** of transformations (**Diffeomorphisms**)

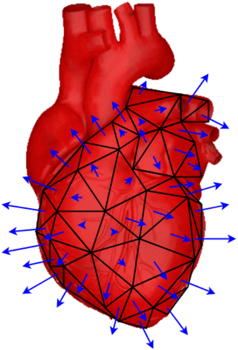
Shape inputs



(a) 3D heart shape



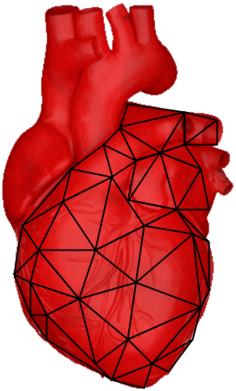
(b) Meshed shape



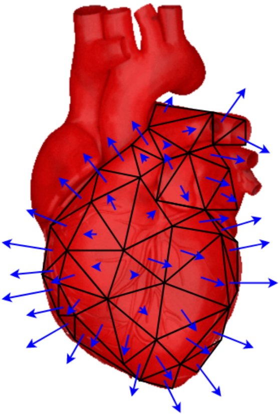
(c) Directional shape

Goal : Compression of (directional) signals via a non-linear network M.

Natural shape transformations

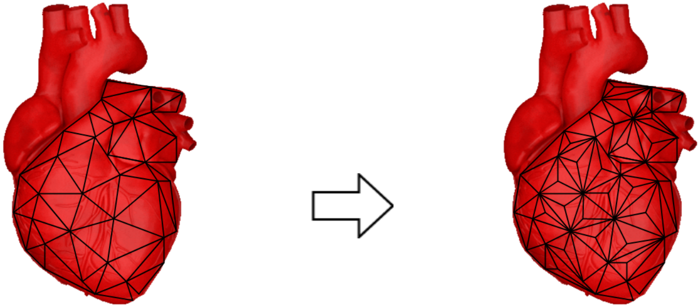


(a) Shape



(b) Directional Shape

Infinite mesh refinements

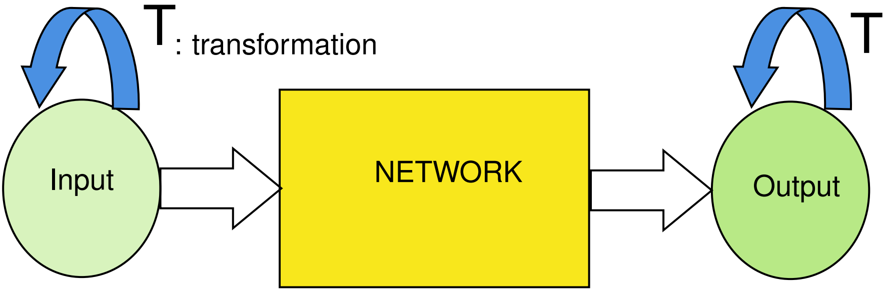


(a) Initial mesh (b) Barycenter refined mesh

In the limit, **Diffeomorphisms** describe natural shape transformations

* it is the **biggest possible** space of transformations

What is equivariance?



Equivariance

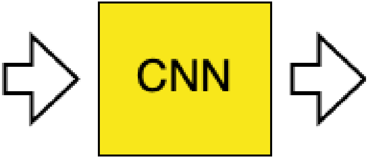
Why look for equivariant networks?

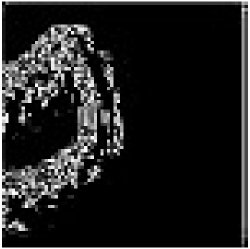
- Transformations on **compressed** shapes (e.g. helps reconstruction)
- **Inductive bias**

- **Reduce complexity** of network
- **Increases accuracy** for task related to the tansformations (e.g. invariance).

Equivariance to translations

(a) Input image



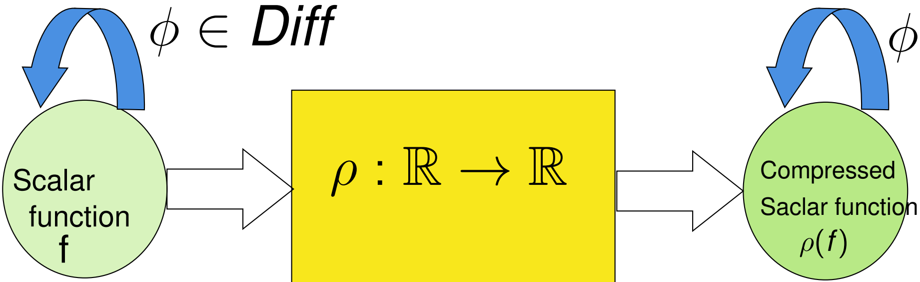


(b) ResNet output

CNN are equivariant to translations

Restrictive result on shape diffeomorphism equivariance

- **Equivariant Networks** to diffeomorphism on **shapes** are ponctual non-linearities $\rho : \mathbb{R} \to \mathbb{R}$



Equivariance to *Diff*: **Scalar case**

Restrictive result on directional shape diffeomorphism equivariance

- **Equivariant Networks** to diffeomorphism on **directional shapes** are multiplications by a constant $\lambda \in \mathbb{R}$

Equivariance to *Diff*: **Vector case**

Conclusion: limitations of deep learning for shape analysis

We showed that there are no straightforward ‘good’ deep learning networks for shape analysis.

Many thanks

Thank you for your attention.

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